8 Relations

51. Relations can be presented explicitly by listing all ordered pairs they consist of. For example, for $A = \{1, 2, 3, 4\}$, one possible relation on A is:

$$R = \{(1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,2), (4,4)\}.$$

(i) Write the implicit form of the relation R. (What conditions must hold for $x \in A$ to be related to $y \in A$ under R?)

(ii) Is R a reflexive relation?

(iii) Is R a symmetric relation?

(iv) Is R a transitive relation?

52. (1st Exam, November 2021.) On the power set $\mathcal{P}(M)$ of the set $M = \{1, 2\}$, we introduce a relation R defined by:

$$ARB \qquad \Longleftrightarrow \qquad A \cup \{1\} = B \cup \{2\}.$$

List all ordered pairs in R.

53. Let $A = \{4, 5, 6, 7, 8\}$ with the following relation:

$$R = \{(a, b) \in A \times A \mid b = a + 2\}.$$

(a) List all ordered pairs in R.

(b) Is the relation R transitive?

(c) Is the relation R antisymmetric?

(d) Represent the relation with a $0-1$ matrix (label rows and columns with elements of the set A to indicate which element of A represents each row and column).

54. On the set of natural numbers $\mathbb{N}$, we define a relation $\not|$:

$$a \not| b \qquad \Longleftrightarrow \qquad a \text{ does not divide } b \text{ (i.e., } \exists q, r \in \mathbb{N} : b = aq + r \wedge 1 \leq r < a).$$

(a) List at least five elements of the relation $\not|$.

(b) Is the relation $\not|$ reflexive?

(c) Is the relation $\not|$ symmetric?

(d) Is the relation $\not|$ antisymmetric?

(e) Is the relation $\not|$ transitive?

(f) Let $A = \{4, 5, 6, 7, 8\}$. Represent the relation $\not|$ on the set A using a $0-1$ matrix (label rows and columns with elements of the set A).

55. Let $\mathbb{Z}$ be the set of integers. We have the following relation:

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid 5 \text{ divides } a^3 - b^3\}.$$

(a) List at least five elements of the relation R.

(b) Is the relation R reflexive?

(c) Is the relation R symmetric?

(d) Is the relation R transitive?

56. On the set $A = \{1, 2, \ldots, 18\}$, we define a relation R:

$$xRy \qquad \Leftrightarrow \qquad y - x \text{ is a prime number.}$$

(a) Determine the domain and range of the relation R.

(b) Find the set $\{y \in A \mid 10Ry\}$.

(c) Is the relation R symmetric?

(d) Is the relation R transitive?

All above math problems are taken from the following website:
https://osebje.famnit.upr.si/~penjic/teaching.html.
The reader can find all solutions to the given problems on the same page.